GAP Exercise
Computing faithful representations

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Step 1:
Devise and implement an algorithm that, given a finite group G, determines the minimal dimension $\varphi(G)$ of a faithful complex representation of G. Hints:

(a) Read in the literature (for example Isaacs' book) about an introduction on characters and representations.

(b) Use the command 'CharacterTable' and work how to compute the kernel of a representation for a character.

(c) Write a function in GAP that determines the combinations of characters with trivial kernel.

(d) Find a combination of characters as in (c) with minimal dimension.

Step 2:
For each order o at most 500 determine the minimum and the maximum of the following

- $\{\varphi(G) \mid |G| = o, |G| \text{ abelian}\}$.
- $\{\varphi(G) \mid |G| = o, |G| \text{ nilpotent}\}$.
- $\{\varphi(G) \mid |G| = o, |G| \text{ solvable}\}$.
- $\{\varphi(G) \mid |G| = o, |G| \text{ non-solvable}\}$.

Describe your observations. Hints:

(a) Use the command 'SmallGroup'.

(b) Also check out 'SmallGroupsInformation'

(c) This computation may take a while for some orders (which?). Can you make your implementation of Step 1 effective enough, so that it copes with all orders?

Step 3:
Can you extend some or all of the computations from Step 2 to higher orders? How far can you go?